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Assembling Components with Probabilistic Lead Times

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Abstract. This paper examines a production model in which *N* different types of components are used to assemble a finished product. The components are acquired from various suppliers in lots received at or before the beginning of the assembly process. The lead time between placing and receiving each of the *N* orders is assumed to be a random variable having a known probability density function. Based on recent results regarding the probability distribution and the expectation of the maximum of a set of independent random variables, the mathematical model describing this production/inventory situation is developed. The mathematical model is then used to derive a closed form formula for the optimal solution that minimizes the total production/inventory cost function. Moreover, the reorder point is shown to depend on the probability distribution and expected value of the random variable representing the maximum among the *N* lead times. The case in which each of the *N* lead times follows a Weibull distribution is investigated, and a numerical example is given to illustrate this case.

1. Introduction

The classical economic production quantity (EPQ) model determines the optimal production lot size that minimizes the total production/inventory cost by balancing the holding and setup costs. It is based on simplifying and unrealistic assumptions. By relaxing these assumptions, numerous research studies have been carried out so that the modified EPQ model accounts for factors faced in real life.

The classical EPQ model does not account for the various costs of components/raw material parts used in the production process. Salameh and El-Kassar [1] considered an EPQ that uses a single type of raw material. Salameh and Jaber [2] presented an inventory model in which each lot received from the supplier contains some imperfect items that can be salvaged at a discounted price. Khan et al. [3] presented a survey of articles that extend the model of Salameh and Jaber [2]. Several research studies have studied the effect of quality on the EPQ model (Salameh and Jaber [2]; Khan et al. [3]; Yassine et al. [4]). Most extensions of the EPQ model that account for quality of components/raw material parts considered a production process with a single type of components. Yassine [5] investigated a production process that uses different types of components and examined the effect of the quality of the components as well as the emissions tax incurred from transportation. Yassine [5] showed that the optimal production quantity depends on the expected value of the minimum of a set of random variables related to the percentages of perfect quality components of the various types.

This paper examines a production model that uses *N* different types of components in the assembly of the finished product. The components are acquired from various suppliers in lots received at or before the beginning of the assembly process. For each type of components, the lead time between placing and receiving the order is assumed to be a random variable having a known probability density function. A mathematical model describing this production/inventory situation is developed by

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applying results similar to those used in Yassine [6] and Yassine [5]. A closed form formula for the optimal solution is derived based on minimizing the total production/inventory cost function. Moreover, the reorder point is shown to depend on the probability distribution and expectation of the random variable representing the maximum among the lead times. The particular case where each of the lead times has a Weibull distribution is examined. Numerical examples are given to illustrate the calculation of the optimal solution and the reorder point.

2. Mathematical Model

In this section, the mathematical model is developed and the optimal order quantity is derived by minimizing the total inventory cost per unit time function.

2.1. Notation

The following notation will be used:

D	Demand rate of the finished product (units/unit time)
P	Assembly rate (units/unit time)
y	Ordering quantity of the finished product (units)
y^*	Optimal ordering quantity of the finished product (units)
C	Cost of assembling one finished product ($/unit)
C_0	Set up cost of assembly ($)
C_i	Purchasing cost per component of type i ($/unit)
K_i	Order cost of components of type i ($)
h_i	Holding cost of a component of type i per unit time ($/unit/unit time)
h	Holding cost of one finished product per unit time ($/unit/unit time)
T	Length of the inventory cycle (unit time)
T_p	Length of the assembly period (unit time)
L_i	Lead time of components of type i (unit time)
$f_i(L_i)$	Probability density function of L_i
μ_i	Expected value of L_i (unit time)
σ_i	Standard deviation of L_i (unit time)
L	Lead time between placing and receiving all components (unit time)
μ_L	Expected lead time of L (unit time)
R	Reorder point (units)
$1-\alpha$	Service level
SS	Safety stock of finished items (units)

2.2. Assumptions

The development of the mathematical model is based on the following assumptions:

- The finished product is assembled using N types of components.
- A single item of the finished product requires one unit of each type of components.
- All components received from the suppliers contain some imperfect quality items.
- The finished product demand rate D is deterministic.
- Assembly time is significant.
- The lead time L_i for each type of components is a random variable having a known probability distribution $f_i(L_i)$.
- An order of size y of each type of components is placed whenever the inventory level of the finished reaches a reorder point R.

2.3. The Total Cost per Unit Time Function

The objective of this model is to determine the optimal quantity y^* of the finished product as well as the reorder point R. The lead time L is defined to be the time between placing and receiving the N orders is given by

$$L = \max(L_1, L_2, ..., L_N). \quad (1)$$

Given a required service level of (1-α)×100%, the reorder point R is determined by

$$P(LD > R) = \alpha. \tag{2}$$

This reorder point R is the sum of the expected demand during lead time, $\mu_L D$, and the safety stock, SS, that guaranties the required service level of (1-α)×100%. That is,

$$R = \mu_L D + SS. \tag{3}$$

The total cost per inventory cycle function, $TC(y)$, is the sum of the setup cost of assembly, the costs of ordering the components, as well as the holding costs of components, finished product, and the safety stock. From Figure 1, the $TC(y)$ function is given by

$$TC(y) = C_0 + Cy + \sum_{i=1}^{N}(C_i y + K_i) + \frac{h}{2}(P - D)T_p T + h(R - \mu_L D)T + \sum_{i=1}^{N} h_i\left((L - L_i)y + \frac{y}{2}T_p\right). \tag{4}$$

The total cost per unit time function, $TCU(y)$, is obtained by dividing the expression in equation (4) by the inventory cycle length T. The expected total cost per unit time function $ETCU(y) = E(TC(y)/T)$. By the Renewal Reward Theorem, $ETCU(y)$ is approximated by $E(TC(y))/E(T)$. Since the order quantity is y, the expected cycle length is y/D and the expected production period is y/P. Hence, $E(T) = y/D$, $E(T_p) = y/P$, and the expected total cost per unit time function $ETCU(y)$ is

$$ETCU(y) = \left(\frac{C_0}{y} + C\right)D + \sum_{i=1}^{N} D\left(C_i + \frac{K_i}{y}\right) + \frac{yh}{2}\left(1 - \frac{D}{P}\right) + h(R - \mu_L D) + \sum_{i=1}^{N} Dh_i\left((\mu_L - \mu_i) + \frac{y}{2P}\right). \tag{5}$$

Differentiating $ETCU(y)$,

$$\frac{d}{dy}\left(ETCU(y)\right) = -\frac{C_0 D}{y^2} - \sum_{i=1}^{N}\frac{K_i D}{y^2} + \frac{h}{2}\left(1 - \frac{D}{P}\right) + \sum_{i=1}^{N} h_i \frac{D}{2P}. \tag{6}$$

Setting the derivative in equation (6) equal to zero and solving for y, the optimal order quantity of the finished product y^* is

$$y^* = \left(\frac{C_0 D + \sum_{i=1}^{N} K_i D}{\frac{h}{2}\left(1 - \frac{D}{P}\right) + \sum_{i=1}^{N} h_i \frac{D}{2P}}\right)^{1/2}. \tag{7}$$

Note that the second derivative of $ETCU(y)$, obtained by differentiating the equation (6), is always negative. Hence, y^* is the unique minimizer of $ETCU(y)$.

3. Lead Times with Weibull Distributions

Among the probability distributions identified in the literature for the lead time is the Weibull distribution [7].

Let L_1 and L_2 be two independent random variables having Weibull distributions with positive scale parameters α_1 and α_2 and identical positive shape parameters each equal to k. It can be shown that the probability density function of $L = max(L_1, L_2)$ is

$$f_L(l) = \frac{k}{\alpha_1}\left(\frac{l}{\alpha_1}\right)^{k-1}.\exp\left(-\left(\frac{l}{\alpha_1}\right)^k\right) + \frac{k}{\alpha_2}\left(\frac{l}{\alpha_2}\right)^{k-1}.\exp\left(-\left(\frac{l}{\alpha_2}\right)^k\right) - \frac{k}{\alpha^*}\left(\frac{l}{\alpha^*}\right)^{k-1}.\exp\left(-\left(\frac{l}{\alpha^*}\right)^k\right) \tag{8}$$

where $x \geq 0$ and $\alpha^* = \left(\frac{1}{\alpha_1^k} + \frac{1}{\alpha_2^k} \right)^{-\frac{1}{k}}$. Moreover, cumulative distribution of L is

$$F_L(t) = 1 - .\exp\left(-\left(\frac{l}{\alpha_1} \right)^k \right) - \exp\left(-\left(\frac{l}{\alpha_2} \right)^k \right) + \exp\left(-\left(\frac{1}{\alpha_1^k} + \frac{1}{\alpha_2^k} \right) l^k \right), \quad (9)$$

and the expected value of L is

$$\mu_L = (\alpha_1 + \alpha_2 - \alpha^*)\Gamma\left(1 + \frac{1}{k} \right)^{k-1}. \quad (10)$$

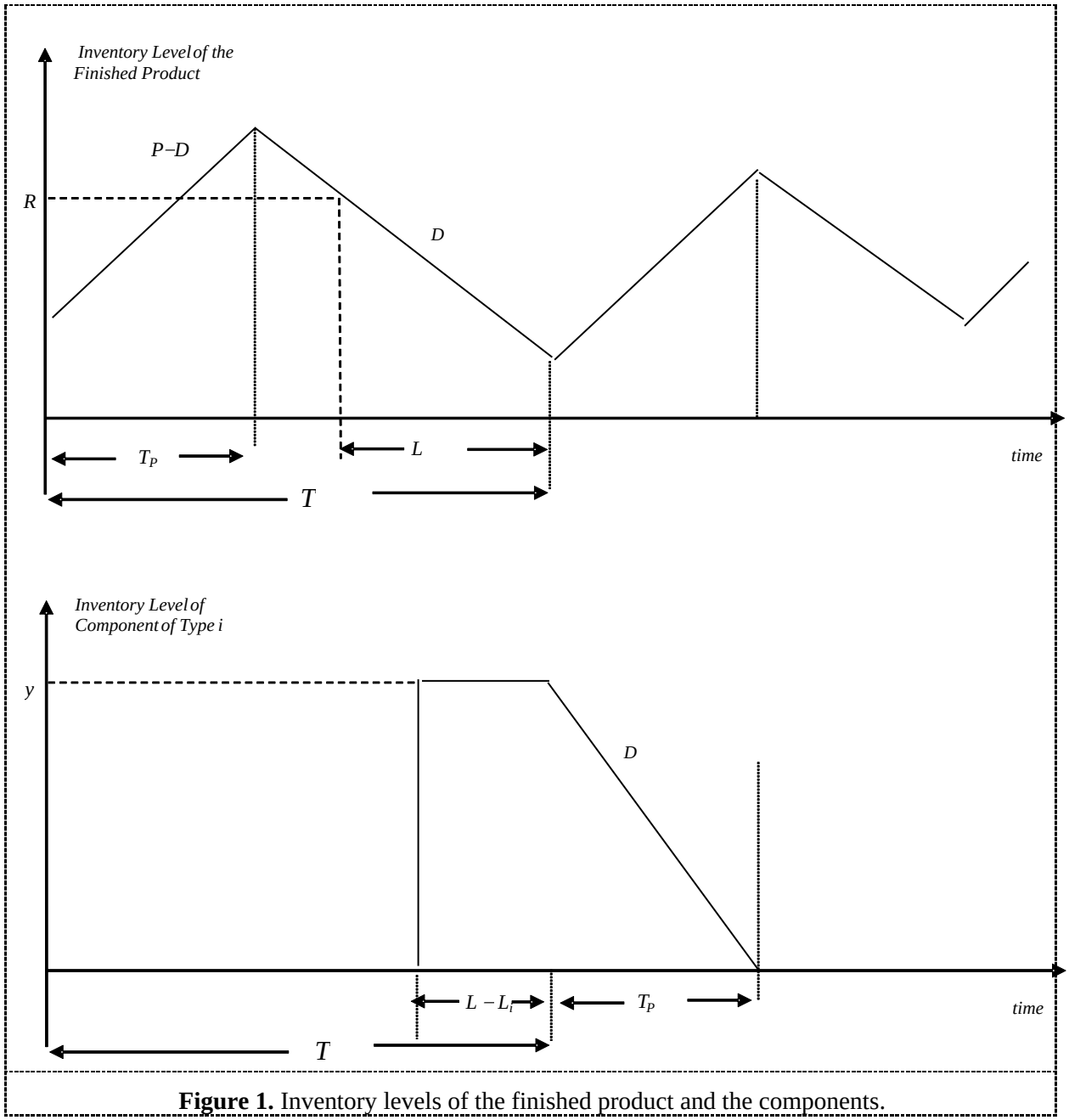

Figure 1. Inventory levels of the finished product and the components.

4. Numerical Example

Suppose that the demand rate of the finished product is 200 units per day. A single item of each of the two types of components, 1 and 2, are assembled to form a finished item. The costs of ordering components of type 1 and type 2 are \$200 and \$300, respectively. The unit purchasing costs of components of type 1 and type 2 are \$2 and \$3, respectively. The holding cost per unit per day is \$0.01 for component 1, \$0.02 for component 2, and \$0.06 for the finished product. The assembly cost is \$4 per item of the finished product. The assembly rate is 400 units/day. The optimal order quantity obtained from equation (6) is y^*= 3,651 units.

Suppose that the lead times in months follow Weibull distributions with parameters $\alpha_1 = \alpha_2 = k = 1$. Then, the expected lead times μ_1 and μ_2 are both equal to 1 month. From equation (9), the expected value of L is μ_L = 1.499 months. Substituting the parameters of the distributions in equation (8), the cumulative distribution of L is $F_L(t) = 1 - 3e^{-t}$. For a service level of 95%, the reorder point obtained from equation (2) is R = 800 units. The safety stock calculated using equation (3) is SS = 500 units. The minimum expected total cost per day is \$1,998.40.

5. References

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