The conformable reduced differential transform method for solving Newell-Whitehead-Segel Equation with non-integer order

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The conformable reduced differential transform method for solving Newell-Whitehead-Segel Equation with non-integer order

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Abstract. In this study, we aim at solving analytically and approximately the conformable fractional Newell-Whitehead-Segel equation (CFNWSE) via conformable reduced differential transform method (CRDTM). Through utilizing the proposed procedure, CFNWSE is converted to an iterative expression which can be easily solved using the initial condition.

Four numerical examples, which we already knew their exact solution using other numerical methods, were solved by CRDTM to examine the competence of this method in solving the CFNWSEs. It is observed that CRDTM gives solutions that coincide with the exact solutions, and it saves a lot of computational work in solving FNWSEs. Moreover, the CRDTM is an efficient and simple tool for dealing with the CFNWSEs.

AMS Subject Classification (2010): 34A08, 35A20, 35C05

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1. Introduction

On September 30, 1695 fractional calculus was introduced when Leibniz wrote a letter to L'Hopital, questioning "what does it mean by $\frac{dx^n}{dx^n}$ when $n = \frac{1}{2}$? " where $\frac{d^n f}{dx^n}$ is known as the nth-derivative of the map f(x) [1-4] .

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Years later and in the attempt to answer L'Hopital's question, the study of non-integer order derivatives became the center of attention of many researchers who tried to set definitions for fractional derivatives [5]. Several forms of fractional (non-integer) derivatives were introduced-Riemann-Liouville, Hadamard, Coimbra, Caputo, Riesz ,Marchaud and Canavati are just a very few of numerous other definitions [6, 7]. And as this new fractional calculus came to light, many fractional partial differential equations (FPDEs) unfolded in the process.

Furthermore, FPDEs turned out to play a substantial role in modelling countless phenomena that arise in applied sciences. Researchers have used numerical and analytical methods to solve these equations.

However, it was noticed that exact solutions for those FPDEs seldom existed, and obtaining numerical and analytical solutions for them was not an easy task, it was rather a very demanding one. Therefore, too many investigators have attempted to set a suitable method (numerical or analytical) for finding solutions to the complicated FPDEs involving nonlinear terms.

The well-known method named RDTM has been widely used by a huge number of studies for solving such kind of equations. For instance, Momani et al. [8], Jafari et al. [9], Thabet and Kendre [10], Yang et al. [11], Jneid and Chakik [12] , Acan et al. [13], Singh and Kumar [14], Chaouk and Jneid [21] and the references cited therein, they obtained solution of FPDEs by using RDTM.

In this work, we are concerned with solving the following conformable fractional NWSE:

$$\frac{\partial^{\alpha}}{\partial t^{\alpha}}u(x,t) = k\frac{\partial^{2\beta}}{\partial x^{2\beta}}u(x,t) + au(x,t) - bu^{q}(x,t),$$

where $0 < \alpha, \beta \leq 1$; $a, b \in \mathbf{R}$ and $k, q \in \mathbf{Z}^{+}$.

In 1969 Newell, Whitehead and Segel derived this equation [15, 16], and ever since then it has been employed to model many nonlinear problems appearing in fluid mechanics. NWSE have huge applicability in ecology, biology, chemistry, engineering and so many other areas of science.

In the previous years, many researchers have suggested different methods for solving this equation. In 2013, Ezatti and Shakibi attempted to solve two nonlinear NWS equations. They did that by applying Adomian decomposition method and the reduced differential transform method [17].

We apply here the CRDTM as a new technique to obtain the approximate and analytical solution of the CFNWSE.

2. On conformable derivative

We first present the two most commonly used fractional derivatives in different applications:

Definition 2.1 [3]. Riemann-Liouville's fractional derivative of a suitable mapping h is given as

$$ {}_{RL}^{\alpha}D_y h(y) = \frac{d^m}{dy^m}\int_0^y \frac{1}{\Gamma(m-\alpha)}(y-s)^{m-\alpha-1} h(s)ds \qquad m-1 \le \alpha < m $$

where α is the order of derivative of h and m $\in \mathbf{Z}^+$.

Definition 2.2 [3]. Caputo fractional derivative of a suitable mapping h is given as

$$ {}_{Cp}^{\alpha}D_y h(y) = \int_0^y \frac{1}{\Gamma(m-\alpha)}(y-s)^{m-\alpha-1} \frac{d^m h}{ds^m}ds \qquad m-1 \le \alpha < m $$

where α is the order of derivative of h and m $\in \mathbf{Z}^+$.

The use of integrals in the former definitions of fractional derivatives creates some major inconsistencies that limit the extent of their use in several applications. For instance, classical properties of calculus (Quotient rule, Mean Value Theorem, Rolles Theorem) are not satisfied in almost all fractional derivatives.

To overcome these complications, Khalil et al. [18] lately proposed an innovative fractional derivative in 2014, named conformable fractional derivative(CFD).

This new definition is very similar to the classical derivative. It depends upon the basic limit definition as shown in Definition 3, and consequently allows the easy extension of some typical theorems in calculus that the existing definitions of fractional derivatives did not allow, due to its simple nature.

For more details of the following definition and theorem, see [18, 19].

Definition 2.3 [18, 19]. Given a mapping $h : \mathbf{R} \times [0, \infty\ ) \to \mathbf{R}$, the partial CFD of $h(x,t)$ of non-integer order α is defined as

$$\frac{\partial^\alpha}{\partial t^\alpha} h(x,t) = \lim_{\varepsilon \to 0} \frac{h(x, t + \varepsilon t^{1-\alpha}) - h(x,t)}{\varepsilon}$$

for all $t > 0$, $\alpha \in (0, 1]$.

If this limit exists, h is said to be partially α-differentiable at $t > 0$.

Note that we will refer to the notation $\frac{\partial^\alpha}{\partial t^\alpha} f(x,t)$ as ∂_t^α f (x, t) for brevity, and in all of the following $\alpha \in$ (0, 1].

Theorem 2.4 [18, 19]. *Let $f, g : \mathbf{R} \times [0, \infty\ ) \to \mathbf{R}$, be partially α-differentiable at a point $t > 0$ and c, d be real numbers. Then,*

(i) $\partial_t^\alpha (cf + dg) = c\partial_t^\alpha f + d\partial_t^\alpha g$

(ii) $\partial_t^\alpha (t^c) = ct^{t-\alpha}$

(iii) $\partial_t^\alpha f(x,t) = 0$, if $f(x,t)$ was a function depending only on x.

(iv) $\partial_t^\alpha (f/g) = \dfrac{g\partial_t^\alpha (f) - f\partial_t^\alpha (g)}{g^2}$

(v) $\partial_t^\alpha(fg) = f\partial_t^\alpha(g) + g\ \partial_t^\alpha(f)$

(vi) If $f(x,t)$ is partially α-differentiable with respect to t then,

$$\partial_t^\alpha(f(x,t)) = t^{1-\alpha}\partial_t^\alpha f(x,t)$$

3. Conformable reduced differential transform method (CRDTM)

Some basic definitions and theorems related to the CFRDTM are briefly reviewed in this section. For more details, we refer to [11, 20] and the references therein.

Definition 3.1. Assume that $u(x,t)$ is infinitely partially α-differentiable around zero with respect to t then, CFRDT of u(x,t) is given as

$$U_k^\alpha(x) = \frac{1}{\alpha^k k!}\left[(\partial_t^\alpha)^k u(x,t)\right]_{t=0},$$

where $(\partial_t^\alpha)^k u(x,t) = \underbrace{(\partial_t^\alpha\partial_t^\alpha.....\partial_t^\alpha)}_{k \text{ times}} u(x,t)$, and k is a non-negative integer.

Definition 3.2. Given that $U_k^\alpha(x)$ is the CFRDT of $u(x,t)$.Then the inverse CFRDT of $U_k^\alpha(x)$ is given by

$$u(x,t) = \sum_{k=0}^{\infty} U_k^\alpha t^{\alpha k},$$

where the CFRDT of $u(x,t)$ at the initial conditions is defined as :

$$U_k^\alpha(x) = \begin{cases} \dfrac{1}{(\alpha k)!}\left[(\partial_t^{\alpha k} u(x,t)\right]_{t=t_0} & if\ k\ \alpha \in \mathbf{Z}^+ \text{ for } k = 0,1,...\left(\dfrac{n}{\alpha}-1\right) \\ 0 & \text{if } \ k\ \alpha \notin \mathbf{Z}^+ \end{cases}$$

such that n represents the order of CFPDE.

By consideration of $U_0^\alpha(x) = f(x)$ as transformation of the I.C. $u(x,0) = f(x)$ Straightforward iterative calculations gives the $U_k^\alpha(x)$ values for $k = 1,2,3,...,n$.

Then the inverse transformation of the $U_k^\alpha(x)_{k=0}^n$ gives the approximate solution as

$$\psi_n(x,t) = \sum_{k=0}^{n} U_k^\alpha(x) t^{\alpha k}$$

where n represents the order of the obtained approximation solution.

Hence, the CFRDTM leads a solution as follows:

$$u(x,t) = \lim_{n\to\infty} \psi_n(x,t) \ .$$

Fundamental operations of CFRDT are displayed in the theorem below [20]:

Theorem 3.3 [20]. *Let $u(x,t)$, $v(x,t)$ and $w(x,t)$: $\mathbf{R}^n \times [0,\infty) \to \mathbf{R}$, be partially α-differentiable at a point $t > 0$ and $a, b \in \mathbf{R}$. Then,*

(i) *If $u(x,t) = v(x,t).w(x,t)$ then $U_k^\alpha(x) = \sum_{s=0}^{k} V_k^\alpha(x,s) W_{k-s}^\alpha$*

(ii) If $v(x,t) = \partial_t^\alpha u(x,t)$ then $V_k^\alpha(x) = \alpha(k+1) U_{k+1}^\alpha(x)$.

(iii) If $u(x,t) = av(x,t) \pm bw(x,t)$ then $U_k^\alpha(x) = aV_k^\alpha(x) \pm bW_k^\alpha(x)$.

In general for $u(x,t) = v_1(x,t)v_2(x,t)...v_n(x,t)$ then we have,

$$U_k^\alpha(x) = \sum_{k_{n-1}=0}^{k_n} ... \sum_{k_1=0}^{k_2} V_{(1)k_1}^\alpha V_{(2)k_2-k_1}^\alpha \times ... V_{(n-1)k_{n-1}}^\alpha V_{(n)k_n-k_{n-1}}^\alpha$$

(iv) If $u(x,t) = t^m h(x)$ then

$$U_k^\alpha(x) = \delta(k - \frac{m}{\alpha}) h(x),$$

where

$$\delta(k) = \begin{cases} 1 \ if \ k = 0 \\ 0 \ if \ k \neq 0 \end{cases}$$

4. CRDTM to solve the FNWSEs:

The standard form of CFNWSE is expressed as follows:

$$\partial_t^{\alpha} u(x,t) = k\partial_x^{2\beta} u(x,t) + au(x,t) - bu^q(x,t), \tag{1}$$

where $x \in \mathbf{R}, t > 0,\ 0 < \alpha, \beta \leq 1$, constrained to I.C.:

$$u(x,0) = f(x) \tag{2}$$

By taking the conformable fractional reduced differential transform (CRDT) of both sides of equation (1) and using the elementary operations of CFRDTM, we obtain:

$$\begin{aligned} \alpha(k+1)U_{k+1}^{\alpha}(x) &= \partial_x^{2\beta} U_k^{\alpha}(x) + aU_k^{\alpha}(x) \\ &\quad -b \sum_{k_{q-1}}^{k} \sum_{k_{q-2}=0}^{k_{q-1}} \cdots \sum_{k_1=0}^{k_2} (U)_{k_1}^{\alpha} (U)_{k_2 - k_1}^{\alpha} (U)_{k-k_{q-1}}^{\alpha} \end{aligned} \tag{3}$$

where $U_k^{\alpha}(x)$ is the conformable fractional reduced differential transform function of u(x,t).

Considering $U_0^{\alpha}(x) = f(x)$ and substituting it in (3), we can obtain $U_k^{\alpha}(x)$ for $k = 1, 2, 3...$

Hence the inverse conformable reduced transformation of the $\{U_k^{\alpha}(x)\}_{k=0}^{\infty}$ is given as:

$$u(x,t) = \sum_{k=0}^{\infty} U_k^{\alpha}(x)\ t^{k\alpha} \tag{4}$$

which is the approximate solution of equation (1), (2) by CRDTM.

5. Applications

To check the efficiency of the CRDTM, we consider four examples in this section.

Example 5.1. Consider the following type of CFNWS:

$$\partial_t^{\alpha} u(x,t) = \partial_x^{2\beta} u(x,t) - u^2(x,t), \tag{5}$$

where $0 < \alpha, \beta \leq 1,\ t > 0,\ x \in \mathbf{R}$, constrained to I.C.:

$$u(x,0) = 1 \tag{6}$$

By taking the conformable fractional reduced differential transform (CRDT) of (5), we get:

$$\alpha(k+1)U_{k+1}^{\alpha}(x) = \partial_x^{2\beta} U_k^{\alpha}(x) - \sum_{s=0}^{k} U_{k-s}^{\alpha}(x)\ U_s^{\alpha}(x). \tag{7}$$

where the function $U_k^{\alpha}(x)$ is the conformable fractional reduced differential transform function.

Considering $U_0^{\alpha}(x) = 1$ and substituting it in (7), we get:

$$\begin{aligned} U_1^{\alpha}(x) &= -\frac{1}{\alpha}, \\ U_2^{\alpha}(x) &= \frac{1}{\alpha^2}, \\ U_3^{\alpha}(x) &= -\frac{1}{\alpha^3}, \\ &\cdots \\ U_n^{\alpha}(x) &= \frac{(-1)^n}{\alpha^n} \end{aligned} \tag{8}$$

Hence, inverse transformation of the $\{U_k^{\alpha}(x)\}_{k=0}^{\infty}$ is found as:

$$u(x,t) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\alpha^k}\ t^{k\alpha} = \frac{1}{1 + \frac{t^{\alpha}}{\alpha}}$$

The approximate solution of (5) and (6) by CRDTM is the same as the exact solution.

Example 5.2. Consider the following CFNWSE:

$$\partial_t^{\alpha} u(x,t) = \partial_x^{2\beta} u(x,t) - 2u(x,t) \tag{9}$$

where $0 < \alpha, \beta \leq 1,\ t > 0,\ x \in \mathbf{R}$ constrained to I.C.:

$$u(x,0) = e^{\frac{x^\beta}{\beta}}. \tag{10}$$

By taking the conformable fractional reduced differential transform (CRDT) of (9), we obtain:

$$\alpha(k+1)U_{k+1}^{\alpha}(x) = \partial_x^{2\beta} U_k^{\alpha}(x) - 2U_k^{\alpha}(x) \tag{11}$$

where the function $U_k^{\alpha}(x)$ is the conformable fractional reduced differential transform function.

Considering $U_0^{\alpha}(x) = e^{\frac{x^\beta}{\beta}}$ and substituting it in (11), we obtain:

$$\begin{aligned} U_1^{\alpha}(x) &= \frac{1}{\alpha} e^{\frac{x^\beta}{\beta}} \\ U_2^{\alpha}(x) &= \frac{1}{2\alpha^2} e^{\frac{x^\beta}{\beta}}, \\ &\dots \\ U_n^{\alpha}(x) &= \frac{(-1)^n}{n\alpha^n} e^{\frac{x^\beta}{\beta}} \end{aligned} \tag{12}$$

Hence, inverse transformation of the $\{U_k^{\alpha}(x)\}_{k=0}^{\infty}$ is found as:

$$u(x,t) = \sum_{k=0}^{\infty} e^{\frac{x^\beta}{\beta}} \frac{(-1)^k}{k\alpha^k} t^{k\alpha} = e^{\frac{x^\beta}{\beta} - \frac{t^\alpha}{\alpha}} \tag{13}$$

The approximate solution of (9) and (10) by CRDTM is the same as the exact solution.

Example 5.3. Consider the FCFNWS equation:

$$\partial_t^{\alpha} u(x,t) = \partial_x^{2\beta} u(x,t) - u(x,t), \tag{14}$$

where $0 < \alpha, \beta \leq 1, t > 0, x \in (0, \pi)$. constrained to I.C.:

$$u(x,0) = \sin(\frac{x^\beta}{\beta}), \tag{15}$$

and boundary condition:

$$u(0,t) = 0. \tag{16}$$

By taking the conformable fractional reduced differential transform (CRDT) of (13), we get :

$$\alpha(k+1)U^{\alpha}_{k+1}(x) = \partial^{2\beta}_{x} U^{\alpha}_{k}(x) - U^{\alpha}_{k}(x) \tag{17}$$

where the function $U^{\alpha}_{k}(x)$ is the conformable fractional reduced differential transform function.

Considering $U^{\alpha}_{0}(x) = \sin(\frac{x^{\beta}}{\beta})$ and substituting it in (17), we get:

$$\begin{aligned} U^{\alpha}_{1}(x) &= -\frac{2}{\alpha}\sin(\frac{x^{\beta}}{\beta}), \\ U^{\alpha}_{2}(x) &= \frac{2}{\alpha^2}\sin(\frac{x^{\beta}}{\beta}), \\ U^{\alpha}_{3}(x) &= -\frac{4}{3\alpha^3}\sin(\frac{x^{\beta}}{\beta}), \cdots \end{aligned} \tag{18}$$

Therefore, the inverse transformation of $U^{\alpha}_{k}(x)^{\infty}_{k=0}$ is found as:

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} U^{\alpha}_{k}(x)\ t^{k\alpha} \\ &= \sin(\frac{x^{\beta}}{\beta})\ (1 - \frac{2}{\alpha}t^{\alpha} + \frac{2}{\alpha^2}t^{2\alpha} - \frac{4}{3\alpha^3}t^{3\alpha} + ...) \\ &= \sin(\frac{x^{\beta}}{\beta})e^{\frac{-2t^{\alpha}}{\alpha}} \end{aligned}$$

which is the approximate solution of (14) and (15) by CRDTM. This solution is the same as the exact solution.

Example 5.4. Consider the CFNWS equation:

$$\partial^{\alpha}_{t} u(x,t) = \partial^{2\beta}_{x} u(x,t) + 2u(x,t) - 3u^2(x,t) \tag{19}$$

$0 < \alpha, \beta \leq 1,\ t > 0,\ x \in \mathbf{R}$ constrained to I.C.

$$u(x,0) = 1 \tag{20}$$

By taking the conformable fractional reduced differential transform (CRDT) on both sides of (19), we obtain :

$$\alpha(k+1)U_{k+1}^{\alpha}(x) = \partial_x^{2\beta} U_k^{\alpha}(x) + 2U_k^{\alpha}(x) - 3\sum_{r=0}^{k} U_{k-r}^{\alpha}(x)\ U_r^{\alpha}(x) \tag{21}$$

where the function $U_k^{\alpha}(x)$ is the conformable fractional reduced differential transform function.

Considering $U_0^{\alpha}(x) = 1$ and substituting it in (21), we get:

$$\begin{aligned} U_1^{\alpha}(x) &= -\frac{1}{\alpha}, \\ U_2^{\alpha}(x) &= \frac{2}{\alpha^2}, \\ U_3^{\alpha}(x) &= -\frac{11}{3\alpha^3}\cdots, \end{aligned} \tag{22}$$

Hence, inverse transformation of the $\{U_k^{\alpha}(x)\}_{k=0}^{\infty}$ is given as:

$$u(x,t) = 1 - \frac{1}{\alpha}t^{\alpha} + \frac{2}{\alpha^2}t^{2\alpha} - \frac{11}{3\alpha}t^{3\alpha} + \cdots$$

which is the approximate solution by using CFRDTM.

However, the exact solution of the given equation is

$$u(x.t) = \frac{\frac{-2}{3}e^{\frac{t^{\alpha}}{\alpha}}}{\frac{1}{3}e^{\frac{2t^{\alpha}}{\alpha}}}$$

To compare the approximate and the exact solution, we used Mathematica to graph and illustrate the difference between the two solutions. In order to prove the efficiency of the proposed reduced conformable reduced differential method for solving CFNWSEs, we drew graphs for the numerical solution as well as the exact one for two values of α.

From figure 1,2 we can clearly see that the solution obtained by the CRDTM almost coincides with the exact solution. The result of this example shows that the CRDTM is a simple tool that gives exact solutions with very little computational work.

CRDTM Solution

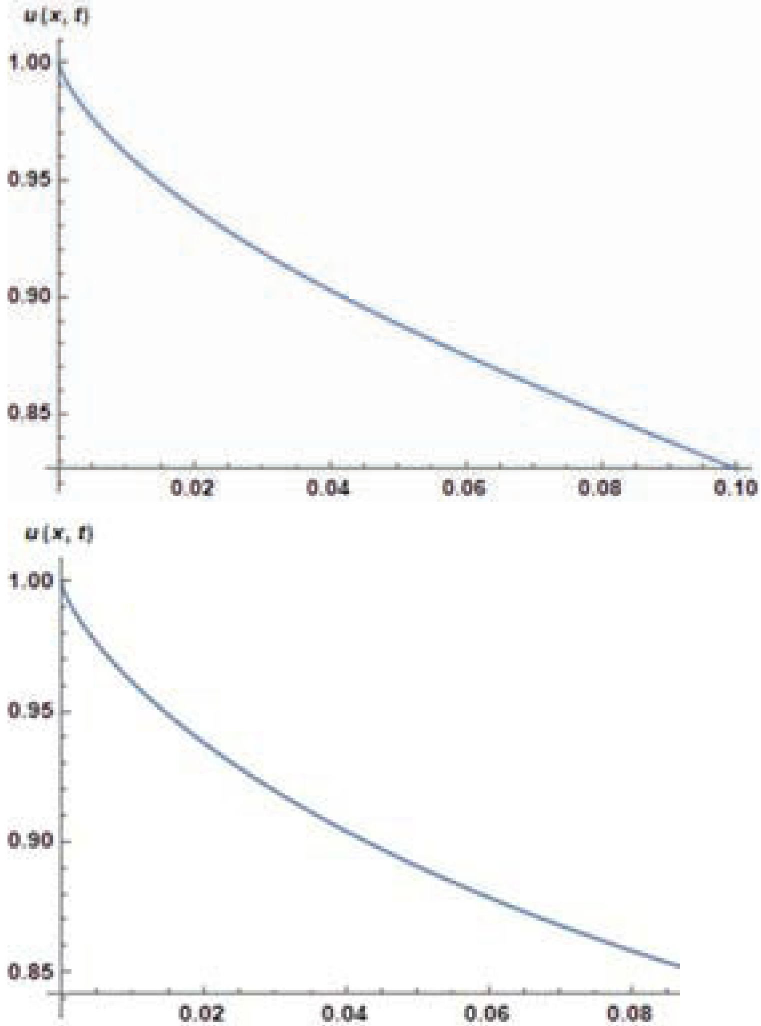

Figure 1: The comparison between the CRDTM solution with the exact solution of Example 4 for $\alpha = 0.75$

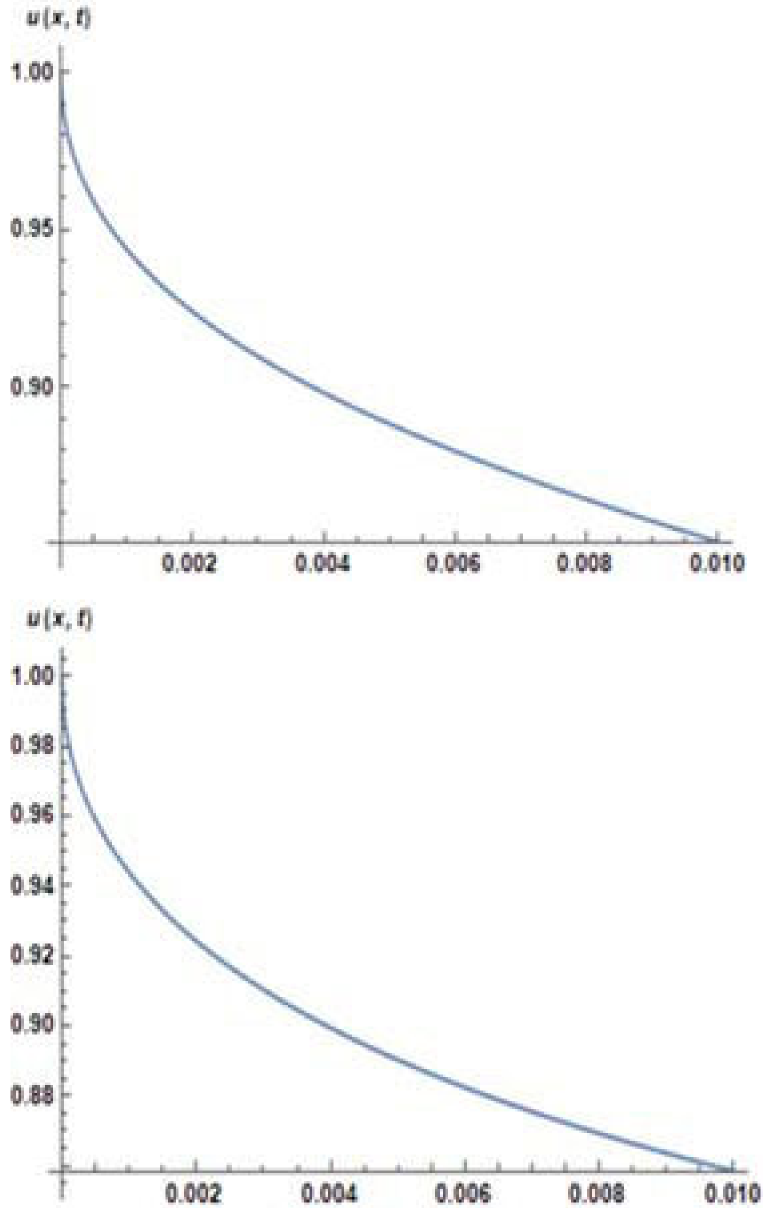

Figure 2: The comparison between the CRDTM solution with the exact solution of Example 4 for $\alpha = 0.5$

6. Conclusion

In this article, the CFRDTM has been employed for solving the conformable fractional Newell-Whitehead-Segel Equation. After dealing with 4 CFNWSEs, the numerical examples gave us series solution which converged very fast to the exact solutions which we knew using other former numerical examples.

Hence, it is concluded that the CFRDTM is a simple and powerful tool for solving CFNWSEs. Not only does it give solutions that agree perfectly with the exact solutions, but with minimal amount of computational work, saving us a lot of time and effort.

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